INVESTIGATION OF THE STATE OF INHOMOGENEOUS MEDIA BY ACOUSTICAL METHODS

A. A. Solov'ev

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The results of measurements of the velocity and absorption of ultrasound in inhomogeneous media are presented. The results are analyzed in relation to two disparate modes of description of inhomogeneous media.

Sound waves are often used for the determination of states of matter that clearly violate the conditions of continuity and homogeneity of the medium [1]. Specific difficulties are met in the interpretation of investigations of this nature. These difficulties stem from the fact that the inhomogeneity of a substance conflicts with the properties of a continuum, which are the foundation of hydrodynamics and acoustics.

There are two methods for solving the stated problem. It is postulated in one method that the wave equation is satisfied for a homogeneous medium that is host to distributed inhomogeneities. The description of the acoustical effects involved reduces to a boundary-value problem. The acoustic absorption mechanism is depicted as an energy loss due to coordinated scattering by elements of the inhomogeneous medium [2].

The second approach calls for the determination of a systematic method by which a substance endowed with inhomogeneous physical properties is replaced by a substance having homogeneous properties [3, 4]. The wave equation in this case is formulated for a homogeneous medium equivalent to the given inhomogeneous medium. Now the acoustic absorption is attributed to macroscopic viscosity and thermal conduction.

It would be interesting to ascertain to what extent the two given approaches are compatible with the data of acoustical measurements in inhomogeneous media. In the present article we give some results of measurements of the ultrasonic velocity and absorption in two inhomogeneous media, one a colloidal solution of gelatin in water and the other a two-phase system comparising lycopodium particles in an aqueous NaCl solution, and we analyze their correspondence with the above-noted methods of describing an inhomogeneous medium.

The velocity measurements were carried out on an optical apparatus by an interferometric method [5]. The error of the velocity measurements was 1.2%. The absorption measurements were conducted by the diffraction method [6]. The intensities of the diffraction peaks were recorded photoelectrically. The maximum error in the absorption measurements did not exceed 12%. The ultrasonic frequency was 11 Mhz.
MHz.

The gelatin–water colloidal solution was prepared in the usual way. Fern spores were used to prepare the two-phase system of lycopodium particles in an aqueous NaCl solution. The particles had an almost spherical shape. The mean particle diameter was 30μ. A stable two-phase system was formed by creating a solution of table salt in water at a density equal to the density of the lycopodium particles ($\rho = 1.07$ g/cm^3). Wetting of the lycopodium particles with the water was ensured by removal of the wax coating.

The characteristic variable indicating the state of an inhomogeneous medium is its relative occupancy by inhomogeneities. The results of ultrasonic velocity measurements in systems having different particle counts per unit volume given in Fig. 1, in which it is clear that the ratio squared of the ultrasonic velocity g_0 in the pure solvent to the ultrasonic velocity g in the inhomogeneous medium decreases linearly as the

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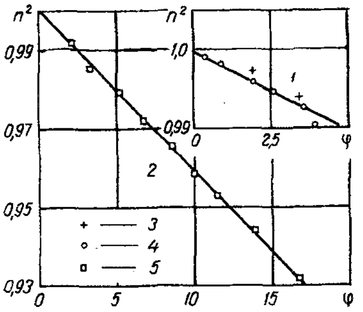

Fig. 1. Refractive index squared versus particle concentration. 1) Gelatin; 2) lycopodium; 3) data of [7]; 4, 5) author's data; φ in %.

concentration is increased. This fact is highly consistent with calculations based on the following equation, which is derived from either of the indicated modes of description:

$$\frac{g_0^2}{g^2} = n^2 = \left(1 + \frac{\rho_i - \rho_0}{\rho_0}\varphi\right)\left[1 + \left(\frac{g_0^2\rho_0^2}{g_i^2\rho_i^2} - 1\right)\varphi\right]. \quad (1)$$

Here ρ_i and g_i are the density of the particles and the corresponding velocity of ultrasound, and ρ_0 is the density of the solvent. The analytical curve shown in Fig. 1 was plotted for a gelatin solution with $\rho_i = 1.3$ g/cm^3 and $g_i = 1375$ m/sec; and for a lycopodium–aqueous NaCl solution system with $\rho_i = \rho_0$ and $g_i = 2090$ m/sec.

The results of our measurements of the ultrasonic absorption coefficient as a function of the particle concentration are shown in Fig. 2a and 2b in the form of the ratio of the absorption coefficient α of the investigated medium to the absorption coefficient α_0 of the solvent. The following general relation has been proposed in [2] for calculation of the acoustic energy loss due to coordinated scattering by elements of the inhomogeneous medium:

$$\alpha = \frac{2}{a\delta}, \quad (2)$$

in which

$$\delta^2 = \frac{\operatorname{tg}\frac{k_0 a}{2} + \frac{k_0^2 a^2}{2\pi}\operatorname{tg}^2\frac{k_0 a}{2}\left(\frac{1}{3}\operatorname{ctg}\gamma + \frac{\pi}{k_0 a}\right)}{-\operatorname{tg}\frac{k_0 a}{2} + \frac{k_0 a^2}{2\pi}\left(\frac{1}{3}\operatorname{ctg}\gamma + \frac{\pi}{k_0 a}\right)}. \quad (3)$$

Here a is the particle separation, $k_0 = 2\pi/\lambda_0$ is the wave number for the solvent, and $\cot\gamma$ is a quantity proportional to the energy radiated by elements of the inhomogeneous medium.

The quantities k_0 and a are equally significant in the given relation. This fact indicates that the acoustic dispersion interval can be entered by varying either the acoustic wavelength or the particle separation. The dispersion relation (3) was found in the approximation $ka < 2\pi$. In order to eliminate the multivalued behavior of the absorption coefficient for weak concentrations in the calculations we use, instead of $k_0 a/2$, the values of $\tan^{-1}(k_0 a/2)$. We calculate the mean particle separations according to the relation $a = r/\varphi$ (where r is the particle radius and φ is their concentration). Agreement with the experimental data for the system lycopodium–aqueous NaCl solution is guaranteed by the following relation for the energy radiated by the particles:

$$\frac{1}{3}\operatorname{ctg}\gamma + \frac{\pi\varphi}{k_0 r} = \frac{k_0 r}{\pi\varphi}. \quad (4)$$

When the particle separation is diminished to definite values the energy radiated by elements of the inhomogeneous medium tends to a minimum. This fact is consistent with the postulated concentration dependence of the indicated quantity [2].

The following equation has been established on the basis of notions developed in [3, 4] for the acoustic dispersion in inhomogeneous media:

$$\frac{\alpha}{\alpha_0} = \frac{\eta(\varphi)\rho_0 g_0^3}{\eta_0 \rho g^3}. \quad (5)$$

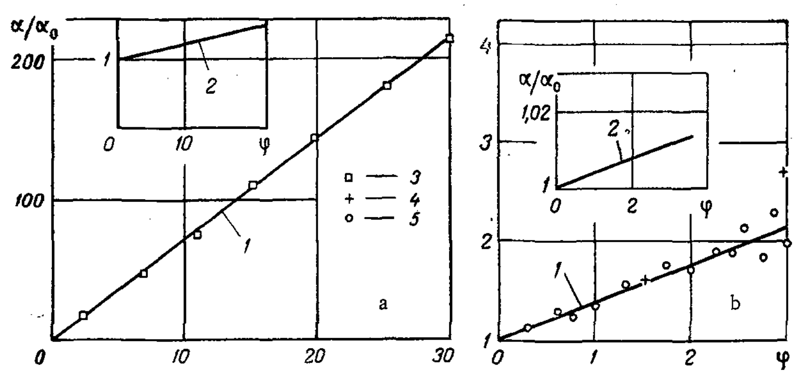

Fig. 2. Relative acoustic attenuation in a the two-phase system lycopodium – aqueous NaCl solution at various particle concentrations (a) and in a gelatin – water solution versus particle concentration (b). 1) Calculated according to Eqs. (3) and (5); 2) calculated from Einstein viscosity equation; 3, 5) author's data; 4) data of [8]; φ in %.

A. S. Predvoditelev has proposed the following relation for the viscosity $\eta(\varphi)$ of an inhomogeneous medium:

$$\eta(\varphi) = \eta_0 \frac{1 + \frac{\varphi}{2r^2 S}}{(1-\varphi)^2}.$$

Here η_0 is the viscosity of the solvent.

The fact that viscosity causes the particles as flow obstacles to have a nonspherical shape is mirrored in the inclusion of the form factor r^2S in the viscosity expression:

$$r^2 S = \frac{r^2 \left[\frac{R_2^3}{R_1^3} \sum \alpha_i^4 + \frac{R_1^3}{R_2^3} \sum \alpha_i'^4 - 3q - 2 \sum \alpha_i^2 \alpha_i'^2 \right]}{R_1 R_2 \left[\frac{R_2^2}{R_1^2} + \frac{R_1^2}{R_2^2} - \sum \alpha_i \alpha_i' \right]}. \tag{6}$$

Here R_1 and R_2 are the radius vectors describing the position of a certain point relative to the source and sink points of the flow obstacle, α_i and α_i' are direction cosines, and $q = R_2^3 - R_1^3 / R_2^3 R_1^3$. Equation (6) goes over to the Einstein relation [3] for $r^2S = 1$. For spherical particles $r^2S = 2$.

We now use the indicated equations to interpret our results. A calculation of the absorption coefficient with the Einstein viscosity yields a large deviation from the measurement data. The form factor calculated from the values for the ultrasonic absorption coefficient turns out to be identical, within the experimental error limits, and equal to 0.0081 for the lycopodium and to 0.018 for the gelatin. A calculation according to Eq. (6) of the ratio of the linear particle dimensions in the direction of wave propagation to the particle radius yields a value of 1.5. This result indicates that the flow around the particles is decidedly nonspherical, a fact that can be attributed to large oscillations of the particles or to the change of shape of the particles in the sound wave. The latter postulate is in good correspondence with the results of N. P. Kasterin.

On the basis of the foregoing results we submit that it is equally justified to describe the acoustic dispersion in inhomogeneous media either on the basis of energy losses due to scattering or from the standpoint of generalizing the hydrodynamical equations to the case of a medium having inhomogeneous physical properties. Consequently, Kasterin's detailed investigation of the dissipation of acoustic energy can be replaced by calculations in which an effective macroscopic viscosity is introduced.

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